S&P 500 RETURNS

Figure 5.9: Largest and smallest returns of S&P 500. The 10 smallest returns are shown with red and the 10 largest returns are shown with blue.

Figure 5.10: Simulated i.i.d. time series. (a) Student's t-distribution with degrees of freedom $\nu = 3$. (b) Student's t-distribution with degrees of freedom $\nu = 6$. (c) Gaussian distribution. The mean of the observations is zero and the standard deviation is equal to the standard deviation of the S&P 500 returns.
Simulated returns

Figure 5.10: *Simulated i.i.d. time series.* (a) Student’s $t$-distribution with degrees of freedom $\nu = 3$. (b) Student’s $t$-distribution with degrees of freedom $\nu = 6$. (c) Gaussian distribution. The mean of the observations is zero and the standard deviation is equal to the standard deviation of the S&P 500 returns-
Figure 5.4: S&P 500 autocorrelation. (a) The sample autocorrelation function $k \mapsto \hat{\rho}(k)$ of S&P 500 returns for $k = 1, \ldots, 1000$. (b) The sample autocorrelation function for absolute returns.
Figure 3.1: *Left and right tail plots.* (a) The left tail plot for S&P 500 returns; (b) the right tail plot. The red curve shows the theoretical Gaussian curve and the blue curves show the Student curves for the degrees of freedom $\nu = 3, 4, 5, 6$. 

The exponential distribution has the density $f(x) = \lambda \exp\{-\lambda x\}$ for $x \geq 0$. The mean is $1/\lambda$ and the variance is $1/\lambda^2$. 

Figure 3.2 shows a smooth left tail plot for the S&P 500 components data, described in Section 2.1.2. Left and right tail plots of GARCH(1,1) and EWMA residuals were shown in Figure xxxx.
Figure 3.3: QQ-plots of the S&P 500 returns. (a) Comparison to a Gaussian distribution and (b) comparison to a Student distribution with degrees of freedom 5 during the period starting at 1950-01-03 and ending at 2009-08-05.
VOLATILITY: MOVING AVERAGES

Figure 3.9
(a) An exponential moving average estimate with \( h = 1 \) of the standard deviation of the S&P 500 index, and (b) an exponential moving average estimate with \( h = 25 \) of the standard deviation of the S&P 500 index.

Figure 3.10
(a) The normalized index values of S&P 500 and Nasdaq-100, and (b) the scatter plot of the net returns.
Figure 6.1: S&P 500 volatility process. The time series of estimated volatility $\hat{\sigma}_t$ in the GARCH(1, 1) model.
Figure 3.9
Volatility of S&P 500
(a) An exponential moving average estimate with $h=1$ of the standard deviation of the S&P 500 index, and (b) an exponential moving average estimate with $h=25$ of the standard deviation of the S&P 500 index.

Figure 3.10
S&P 500 and Nasdaq-100
(a) The normalized index values of S&P 500 and Nasdaq-100, and (b) the scatter plot of the net returns.
Figure 4.1: Scatter plots of the returns of stock indexes S&P 500 and NASDAQ-100 during the period from 1 October 1985 to 17 August 2009.
***CORRELATION PROCESS***

![Graph](image)

**Figure 3.11** S&P 500 and Nasdaq-100; Covariance

The red curve shows a moving average estimator of the covariance between the returns of S&P 500 and Nasdaq-100. The black curve shows the sequentially calculated covariance between the returns of S&P 500 and Nasdaq-100.

#### 3.8.7 Example: Portfolio Selection with Markowitz Criterion

Portfolio choice with mean-variance preferences was proposed by Markowitz (1952) and Markowitz (1959). This method ranks the distributions according to

\[ EU_P - \gamma Var(U_P) \]

where \( \gamma \geq 0 \) is the coefficient of risk aversion and \( U_P = \frac{W_t}{W_{t-1}} - 1 \) is the gross return of the portfolio. We have

\[ EU_P = b^T EU - 1 \]

\[ Var(U_P) = b^T Var(U) b, \]

where \( U = (S_1^t/S_1^{t-1}, ..., S_d^t/S_d^{t-1}) \)

**and** \( Var(U) \) is the \( d \times d \) covariance matrix of \( U \). We have to estimate the vector expected returns \( EU \) and the covariance matrix \( Var(U) \), which can be obtained using the historical data \( S_0, ..., S_t \), \( t = 1, ..., t_0 \).

We can estimate the population moments with the sample versions and to use the arithmetic mean to estimate the expectation and the empirical covariance matrix to estimate the covariance matrix.